Name of Course	: **CBCS B.A. (Prog.)**
Unique Paper Code	: **62351101_OC**
Name of Paper	: **Calculus**
Semester	: **I**
Duration	: **3 hours**
Maximum Marks	: **75 Marks**

Attempt any four questions. All questions carry equal marks.

1. Examine the continuity of the following functions

(i). $f:(0,\infty)\to\mathbb{R}$ defined as

$$f(x)=\begin{cases}7x, & \text{if } 0<x\le 1\\ 2-x, & \text{if } 1<x\le 2\\ x^2-2x, & \text{if } 2<x\le 4\\ x+4, & \text{if } x>4\end{cases}$$

at $x=1,2$ and 4.

(ii). $f:\mathbb{R}\to\mathbb{R}$ defined as

$$f(x)=\begin{cases}\dfrac{5xe^{5/x}}{1+e^{5/x}}, & \text{if } x\neq 0\\ 0, & \text{if } x=0\end{cases}$$

at $x=0$.

(iii). $f:\mathbb{R}\to\mathbb{R}$ defined as

$$f(x)=\begin{cases}-2x^3, & \text{if } x\le 0\\ 5x-8, & \text{if } 0<x\le 1\\ x^2-3x, & \text{if } 1<x<2\\ 3x+4, & \text{if } x\ge 2\end{cases}$$

at $x=0,1$ and $x=2$.

2. If $y=\left[2x+2\sqrt{1+x^2}\right]^m$, then prove that

$$(1+x^2)y_{n+2}+(2n+1)xy_{n+1}+(n^2-m^2)y_n=0.$$

Also, verify Euler's Theorem for

$$z=2^n x^n \log\frac{y}{4x}.$$

3. Find the asymptotes of the following curves

(i). $(x^2-y^2)^2-4x^2+x=0$

(ii). $x^2y+xy^2+xy+y^2+3x=0.$

Also, trace the curve given by

$$y^2(2+x)=6x^2-x^3.$$

4. Verify Lagrange's Mean Value Theorem for the function given by

$$f(x) = (2x-1)(x-3)(2x-5) \text{ for } x \in [0,4]$$

and apply it to prove that

$$\sqrt{1+x} < 1 + \frac{1}{2}x \quad \text{if } x \in (-1, \infty), x \neq 0.$$

Also, verify Cauchy's Mean Value Theorem for following functions

$$f(x) = x^2 - 3x - 5, \qquad g(x) = x^2 + 2x - 1 \text{ in } [0,1].$$

5. Find the equation of the tangent and normal at the point 'θ' to the curve

$$x = 6\cos^3\theta, \quad y = 6\sin^3\theta.$$

Find the radius of curvature at the origin for the following curves:

(i). $x^4 - 4x^3 - 18x^2 - y = 0.$

(ii). $x = 5(\theta + \sin\theta),\ y = 5(1-\cos\theta).$

6. Evaluate

$$\lim_{x\to 0} \frac{1-\cos x^2}{x^2 \sin x^2} \quad \text{and} \quad \lim_{x\to 0^+} (\cot x)^{\sin x}.$$

Find the maximum and minimum values of the function $f(x) = 3x^5 - 15x^4 + 15x^3 - 1$ and separate the intervals in which the function $g(x) = 2x^3 - 9x^2 + 12x - 5$ is increasing or decreasing.